Optimal Variance Swaps

Emanuele Luzzi, Paul Schneider, **Rohan Sen**, Marc Van Uffelen

USI Lugano and Swiss Finance Institute

August 26, 2025

Variance swaps

- Variance swaps quantify the variance risk premia, where the swap rate represents the risk-neutral expected value of the realized return variance.

- A variance swap payoff can be accurately approximated by combining static position in European options with dynamic strategy in the underlying forwards.

$$Z_{t,t+1}(g) = \underbrace{\sum_{k=1}^{n(t)} \left(g'(r_{t,k-1}) - g'(r_{t,0})\right) \pi^{R}_{t,k}}_{\textit{Delta Hedging}} + \underbrace{\sum_{j=1}^{n_K(t)} g''(m_{t,j})\, w_{t,j}\, \pi^{O}_{t,j}}_{\textit{Options Portfolio}}.$$

- The above formula of variance swap payoff depends on a convex function g, which essentially determines the variance risk premium.

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Research goal

- We aim to nonparametrically recover the Sharpe-ratio optimal convex function that can be used to replicate the tradable variance swap contracts from empirical data.

Methodology

- Specify admissible space of nonlinear functions as a Sobolev-type reproducing kernel Hilbert space (RKHS) $\mathcal{H}$ that contains at least twice continuously differentiable functions.

$$\mathcal{H} \subset \mathcal{C}^2(\Omega), \qquad \langle f, g \rangle_{\mathcal{H}} := \langle f, g \rangle_{L^2} + \langle f', g' \rangle_{L^2} + \langle f'', g'' \rangle_{L^2}.$$

- Perform regularized mean-variance optimization in the RKHS to obtain the optimal empirical solution in terms of a finite-dimensional matrix equation.

$$\widehat{g} := \underset{g \in \mathcal{H}}{\operatorname{argmin}} \; -\widehat{\mathbb{E}}[Z_{t,t+1}(g)] + \frac{1}{2}\widehat{\mathbb{V}}[Z_{t,t+1}(g)] + \frac{\lambda}{2}\|g\|_{\mathcal{H}}^2$$

$$\widehat{g} \in \operatorname{span}\left\{ \{r_{t,k-1}\}_{k=1}^{n(t)}, \{m_{t,j}\}_{j=1}^{n_K(t)} : 0 \le t \le T-1 \right\}.$$

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Thank you.

For more details, please visit **Poster 13**.