

Kernel-based Nonparametric Tests for Shape Constraints

Rohan Sen

HKUST Business School, Hong Kong



Motivation: shape restrictions in finance

- ▶ **Pricing kernel:** monotonically decreasing and convex in returns.
- ▶ **Option-implied state price densities:** positivity and convexity.
- ▶ **Equity risk premia and characteristic monotonicity.**
- ▶ **Implied volatility smile:** convexity.
- ▶ **Credit spreads:** positivity and monotonicity in maturity.

These are economic hypotheses, not modeling choices.

Main idea: separate learning from testing

But estimation can hide failures

- ▶ **Standard approach:** impose restriction, estimate, report the shape – violations invisible by construction.
- ▶ **This paper:** Learn a decision rule flexibly, but keep economic structure testable – violations become detectable.

Nonparametric learning $\implies$ Shape inference

What does the theory deliver?

- ▶ **Can we learn what matters for pricing?**

Finite-sample guarantees in the economically relevant payoff norm.

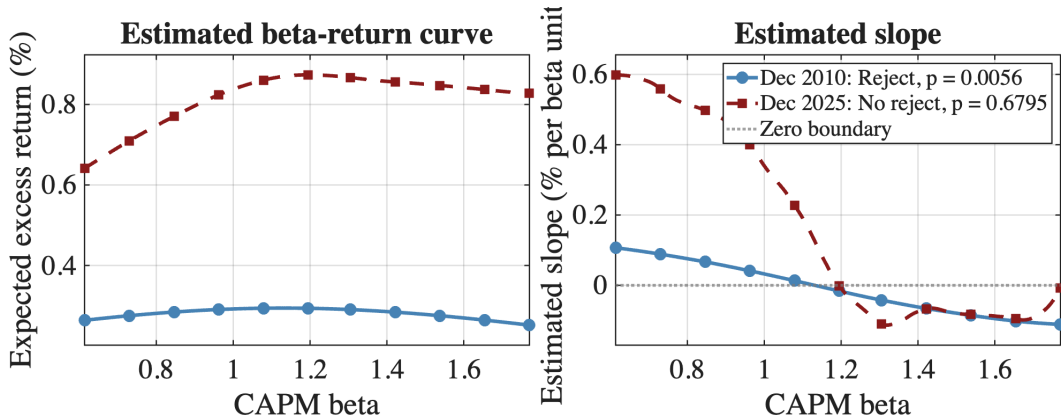
- ▶ **Can we do valid inference with dependent financial data?**

Gaussian inference with HAC long-run covariance.

- ▶ **Can we test economic restrictions without imposing them?**

Cone-projection Wald test with chi-bar-squared calibration, size control, and power against fixed violations.

CAPM: expected excess returns increase with beta



Similar visual nonmonotonicity can imply different statistical evidence.